Internal Conversion in Sn^{119}†

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The total internal conversion coefficient of the ground-state 23.8-keV transition in Sn^{119} and the ratio $[K/(L+M+\cdots)]_{M4}$ of internal conversion coefficients of the preceding 65.3-keV $M4$ transition have been determined by measuring the coincidence rate between the K x rays following internal conversion of the 65.3-keV transition, and the 23.8-keV gamma rays. The following results were obtained: $\alpha=5.2\pm0.3$ and $[K/(L+M+\cdots)]_{M4}=0.42\pm0.02$.

I. INTRODUCTION

THE isotope Sn^{119} has been extensively used in Mössbauer absorption studies. The fractions $f_s=e^{-2W_s}$ of recoilless gamma rays emitted from a crystalline source, or $f_a=e^{-2W_a}$ of recoilless gamma rays absorbed by a crystalline absorber, and therefore the Debye-Waller factor $2W$ for the crystal can be obtained directly from the observed transmission spectra. However, in most cases, it is the quantity $\sigma_0 f_a/(1+\alpha)$ for the absorber which is obtained from the measurements and not just f_a. σ_0 is the maximum resonance absorption cross section and α is the total conversion coefficient for the transition. It is therefore essential to know α as precisely as possible before an accurate determination of the Debye-Waller factor $2W$, and consequently, of the Debye temperature Θ can be made.

Two previous determinations of α, one from measurements with proportional counters of the coincidence rate between the 25.3-keV x rays following internal conversion in the K shell of the 65.3 keV, $M4$ transition in Sn^{119m} (Fig. 1) and the 23.8-keV gamma rays, the other from measurements with scintillation counters of the coincidence rate between the gamma ray and the K x rays following the electron capture decay of Sb^{119} yielded values[1] $\alpha=7.3\pm1.7$ and[2] $\alpha=6.3\pm0.4$, respectively. In addition, the ratio of internal conversion coefficients of the $M4$ transition, $K/(L_{III}+M_V)=0.41$,[3] has been determined from Sn^{119m} electron spectra obtained with a 180° magnetic spectrograph and photographic plates; the value of[4] $(K/L)=1.5$ and[5] $(K/L)=0.82$ had been obtained in earlier work, also from conversion electron spectra. This investigation was undertaken as an attempt to resolve these discrepancies.

II. EXPERIMENTAL ARRANGEMENT

The present experiment also consisted in observing the coincidences between K x rays and gamma rays in Sn^{119m} with scintillation detectors. Two NaI(Tl) crystals, 1 mm thick by 1.5 in. diam, with a 0.002-in. aluminum cover, were mounted on RCA 6355 phototubes located at about equal distances from a 0.5-mm-wide by 0.0005-in.-thick, 0.45-μCi, Sn^{119m} source. The photomultiplier outputs were fed to two double-delay-line clipping amplifiers and single-channel pulse-height analyzers set so as to accept the whole unresolved x-ray $+\gamma$-ray line (Fig. 2). The two gate pulses originating at zero crossover time were delay-line shaped and mixed in a coincidence circuit with a resolving time $\tau=0.25$ μsec. The outputs of the coincidence circuit and of the pulse-height analyzers were then fed to a slower coincidence circuit. Part of the measurements were repeated with a coincidence resolving time of 0.17 μsec and the same true coincidence rate was observed.

The ratio of the coincidence rate N_c to the single channels counting rates N_1 and N_2 was measured under the above conditions as well as with a 0.004-in. palladium absorber between the source and counter #1. This absorber acts as a critical absorber for the x rays [μ_x (25.3 keV) = 56.1 cm²/gm (Ref. 6), μ_γ (23.8 keV) = 10.5 cm²/gm (Sec. III)] such that 28% of the γ rays, but only 0.11% of the x rays can still reach that particular counter. The absorber was supported by a $\frac{1}{16}$-in.-

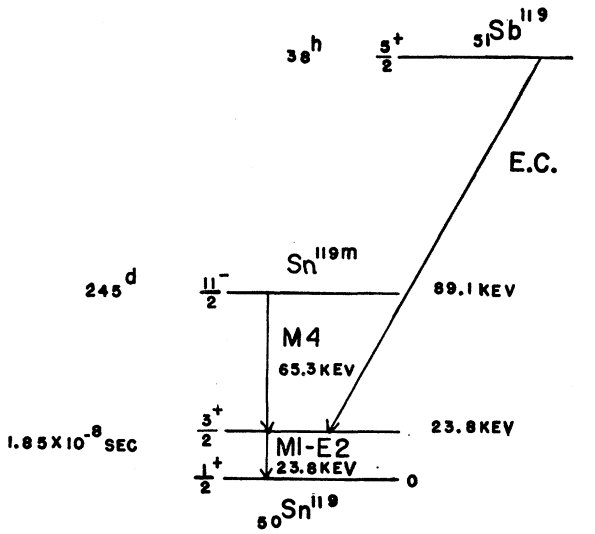

FIG. 1. Energy level diagram of Sn^{119}.

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[2] J. L. Olsen, L. G. Mann, and M. Lindner, Phys. Rev. **105**, 985 (1957).
[3] R. D. Hill, Phys. Rev. **83**, 865 (1951).
[4] J. W. Mihelich and R. D. Hill, Phys. Rev. **79**, 781 (1950).
[5] Nelson, B. H. Ketelle, and G. E. Boyd, ORNL-828, 1950 (unpublished).
[6] R. D. Deslattes, Florida State University, Tallahassee, Florida, AFOSR-TN-58-784-AD-202009 (unpublished).

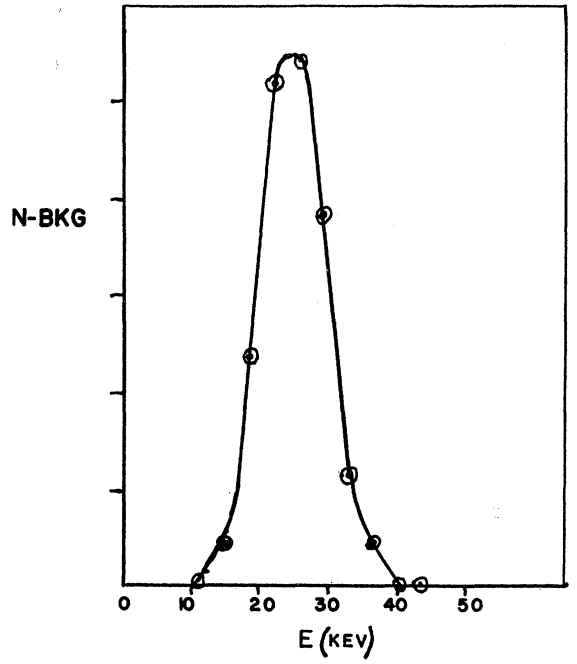

FIG. 2. Sn^{119} scintillation spectrum.

thick lead collimator plate with a 1-cm-diam hole placed 1 cm away from the source. This plate was kept in this position relative to the source during all measurements.

The single and coincidence counting rates for these two geometries can be expressed in terms of ϵ_x and ϵ_γ the counting detection efficiencies for x rays and γ rays, respectively; Ω, the solid angle subtended by the detector at the source; ω_K, the K-shell fluorescence yield for $Z=50$, $R=N_{0x}/N_{0\gamma}$=ratio of x rays to γ rays emitted, $N_{0e}=\alpha N_{0\gamma}$=number of conversion electrons emitted from the 23.8-keV state, and $N_0=N_{0x}+N_{0\gamma}$. No palladium absorber:

$$N_{1,2}=(N_{0x}\epsilon_x+N_{0\gamma}\epsilon_\gamma)\Omega_{1,2}=N_{1,2x}+N_{1,2\gamma},$$

$$N_c=N_{1x}\Omega_2\epsilon_\gamma[N_{0\gamma}/(N_{0\gamma}+N_{0e})]+N_{1\gamma}\Omega_2\epsilon_x[K/(K+L+M+\cdots)]_{M4}\omega_K,$$

$$(N_c/N_1N_2)_0=\omega_K\frac{2(R+1)(\epsilon_\gamma/\epsilon_x)}{N_0(R+\epsilon_\gamma/\epsilon_x)^2[1+(L+M+\cdots)/K]_{M4}}.$$

It has been assumed that $\epsilon_{1x}=\epsilon_{2x}$ and $\epsilon_{1\gamma}=\epsilon_{2\gamma}$.

With palladium absorber it was observed that the counting rate in counter #2 increased by about 2.5% when the palladium absorber was used. These are five contributions to the counting rate of counter #2: direct x and γ rays, x and γ rays Compton scattered backwards by the palladium absorber (about 1.2%), and palladium K x rays of 21.3 keV (about 1.3%). These Pd x rays are created when the Sn x rays are absorbed by photoelectric effect in the Pd K shell, and will unfortunately be unresolved from the two Sn^{119} radiations. The counting rate in counter #1 consists of γ rays and Pd x rays only. Five terms contribute to the coincidence rate: The γ rays will be in coincidence with the direct x rays and Compton scattered x rays as well as with the Pd x rays reaching counter #2, while the Pd x rays will coincide with the direct γ rays and Compton scattered γ rays detected by #2. The first of these five contributions is by far the most important. Since the inclusion of the other four contributions complicates the expression for N_c/N_1N_2 beyond recognition, only the first term has been kept in the formula derived below.

$$N_1=N_{1\gamma}=N_{0\gamma}\epsilon_\gamma\Omega_1e^{-\mu_\gamma\rho t},$$

$$N_2=(N_{0x}\epsilon_x+N_{0\gamma}\epsilon_\gamma)\Omega_2,$$

$$N_c=N_{1\gamma}\Omega_2\epsilon_x[K/(K+L+M+\cdots)]_{M4}\omega_K,$$

$$(N_c/N_1N_2)_{\mathrm{Pd}}=\omega_K\frac{(R+1)}{N_0(R+\epsilon_\gamma/\epsilon_x)[1+(L+M+\cdots)/K]_{M4}}.$$

It then follows that

$$R=(\epsilon_\gamma/\epsilon_x)\left[2\frac{(N_c/N_1N_2)_{\mathrm{Pd}}}{(N_c/N_1N_2)_0}-1\right].$$

The total conversion coefficient α for the 23.8-keV transition and the ratio $[(L+M+\cdots)/K]_{M4}$ for the 65.3-keV transition can be expressed in terms of three experimental quantities, the two coincidences to singles ratios and the effective source strength N_0.

$$\alpha=(R/\omega_K)[1+(L+M+\cdots)/K]_{M4}-1,$$

and

$$[(L+M+\cdots)/K]_{M4}=\omega_K\frac{(R+1)}{N_0(R+\epsilon_\gamma/\epsilon_x)(N_c/N_1N_2)_{\mathrm{Pd}}}-1.$$

The complete expressions have been derived in the Appendix, and were used in the final evaluation of the conversion coefficients. All the corrections lead to less than a 2% change in the results obtained from the above formula.

In order to check these arguments the palladium absorber was replaced by 0.004 in. of silver which produced a 2% increase in the counting rate #2. This increase is due to Compton scattering of x and γ rays only, since silver does not present the critical absorption features of palladium. The ratios $(N_c/N_1N_2)_{\mathrm{Ag}}$ and $(N_c/N_1N_2)_0$ are expected to be identical after the small difference in the absorption coefficients for the x ray and γ rays is taken into account. The measured ratios were $(N_c/N_1N_2)_{\mathrm{Ag}}/(N_c/N_1N_2)_0=1.02\pm0.04$ and 0.99 ± 0.01

TABLE I. Results of the present experiment and comparison with previous measurements and theoretical values. h represents the distance between the source and the detectors. All the given errors are purely statistical except those followed by an asterisk. These include systematic uncertainties.

Geometry	R	$[K/(L+M+\cdots)]_{M4}$	α	Reference
Previous measurements				
			7.3 ±1.7	Bowe and Axe (Ref. 1)
			6.3 ±0.4	Olsen *et al.* (Ref. 2)
		0.41		Hill (Ref. 3)
Present work				
$h=4.0$ cm	1.52±0.03	0.417±0.006	5.11±0.12	coincidence
$h=6.0$ cm	1.56±0.02	0.418±0.004	5.26±0.10	coincidence
$h=9.0$ cm	1.57±0.04	0.422±0.006	5.15±0.19	coincidence
$h=12.0$ cm	1.50±0.05	0.413±0.014	5.12±0.27	coincidence
Average	1.55±0.02	0.42 ±0.02*	5.2 ±0.3*	coincidence
	1.65±0.08*			Critical absorption
	1.65±0.06			Mössbauer absorption
Theoretical				
$M1$	23.8 keV		5.3	
$E2$	23.8 keV		375	Rose (Ref. 10)
$M4$	65.3 keV	0.38		

for source detector distances $h=4$ cm and $h=12$ cm, respectively.

The effect arising from a possible contamination of the source with Sn^{113} was also considered in the Appendix. The 24.2-keV In x rays emitted in the decay of Sn^{113} could not be distinguished from the Sn^{119} gamma rays. However, the isotopic assay, provided by the Oak Ridge Isotope Division with the enriched Sn^{118} sample from which the source was prepared, indicated that no trace of Sn^{112} had been detected at all in the analysis, and a upper limit of 0.05% was given. From this limit it was estimated that by the time the experiment was performed, there should have been less than 2.3% Sn^{113}, if any at all. This amount of impurity would cause an *increase* of 6% in the value of R, and of 20% in $[K/(L+M+\cdots)]_{M4}$ but would *reduce* α by 7%. However, there is evidence from the experiment described in Sec. III that the amount of Sn^{113} contamination in the source is indeed negligible.

The most recent measurement of the fluorescent yield,[7] $\omega_K=0.846\pm0.012$, was used. This value is in good agreement with the empirical value $\omega_K=0.8377\pm0.006$ calculated by Hagedoorn and Wapstra.[8]

N_0 was determined by measuring the counting rate in one of the detectors as a function of the distance h between source and detector. h was varied from 2 to 75 cm, and for $h\geqslant 10$ cm the counting rate was proportional to $1/h^2$. The radius of the NaI(Tl) crystal r is not known very accurately because the edges of the crystal are covered by the container. Therefore, for the purpose of measuring N_0, brass rings $\frac{1}{16}$ in. thick with a 1-in. and a $\frac{3}{4}$-in.-diam hole, respectively, were placed alternately over the crystal, thus clearly defining r and the solid angle. In this case,

$$N_1=(N_{0x}\epsilon_x+N_{0\gamma}\epsilon_\gamma\Omega_1$$
$$=N_{0\gamma}(R\epsilon_x+\epsilon_\gamma)\Omega_1=[N_0/(R+1)](R\epsilon_x+\epsilon_\gamma)\Omega_1,$$

and

$$N_0=\frac{N_1}{\Omega_1}\frac{1}{(R\epsilon_x+\epsilon_\gamma)/(R+1)}.$$

The difference in the values of N_0 determined with and without the brass defining rings was 3%, which can easily be due to the uncertainty in the true size of the crystal, while N_0 obtained with the two different rings agreed to within 1%.

N_0 is inversely proportional to the detector efficiencies, $\epsilon_\gamma=0.966$ and $\epsilon_x=0.964$. These were determined, assuming normal incidence, by calculating the probability that the x rays and γ rays have to be first transmitted by the 0.002-in. aluminum window of the crystal and then absorbed by the 1-mm-thick NaI(Tl) crystal. The absorption coefficients μ_{Al} (23.8 keV) = 1.92 cm^2/gm, μ_{Al} (25.3 keV) = 1.62 cm^2/gm, μ_{NaI} (23.8 keV) = 13.5 cm^2/gm and μ_{NaI} (25.3 keV) = 11.5 cm^2/gm were obtained from the National Bureau of Standards circulars.[9] The estimate of ϵ_x and ϵ_γ might be uncertain by as much as 4%. Hence the main error in N_0 and therefore in α and $[(L+M+\cdots)/K]_{M4}$ stems from these estimates of efficiencies. However, the ratio $\epsilon_\gamma/\epsilon_x$ is likely to be correct to better than 1%, since if there exists a systematic deviation in ϵ_γ, the deviation in ϵ_x will be in the same direction and probably of the same magnitude because the energies of the photons involved are so close, and no sharp discontinuities are

[7] C. E. Roos, Phys. Rev. **105**, 931 (1957).
[8] H. L. Hagedoorn and A. H. Wapstra, Nucl. Phys. **15**, 146 (1960).
[9] G. W. Grodstein, Natl. Bur. Std. (U.S.) Circ. No. 583 (U. S. Government Printing Office, Washington, D. C., 1957); R. T. McGinnies, Natl. Bur. Std. Suppl. Circ. No. 583 (U. S. Government Printing Office, Washington, D. C., 1959).

present at those energies in the absorption coefficients.

To ensure that no systematic errors arose due to yet unaccounted-for scattering or cross talk between detectors, the measurements were carried out for four source-detector positions. In all cases the coincidence rates were corrected for accidentals (true coinc/accidentals=16 with no Pd, =20 with Pd absorber) and the single rates were corrected for background which was measured for each geometry. The results for R, α and $[K/(L+M+\cdots)]_{M4}$ as well as previous measurements and theoretical values,[10] are shown in Table I. The statistical errors in α and $[K/(L+M+\cdots)]_{M4}$ were of the order of 1–5%. The quoted error of 5% reflects the uncertainty in the absolute determination of N_0.

III. SUPPLEMENTARY DETERMINATION OF $N_{0x}/N_{0\gamma}$

There are two alternate methods by which the quantity R might be derived. In one experiment, the ratio of x rays to γ rays emitted by the source is determined from critical absorption measurements with various thicknesses of palladium absorber. These measurements are very sensitive to the exact value of the absorption coefficients μ_γ and μ_x for palladium used in the calculation, as shown below:

$$R=\frac{(e^{-\mu_\gamma\rho t_i}-N^i/N^0)(\epsilon_\gamma/\epsilon_x)}{N^i/N^0-e^{-\mu_x\rho t_i}-(\Omega_{SA}\Omega_{AD}/\Omega_{SD})A_i(\epsilon'/\epsilon_x)},$$

where

$$A_i=\omega_K e^{-\mu'\rho t_i}[1-e^{(-\mu_x+\mu')\rho t_i}]\mu_x{}^K/(\mu_x-\mu').$$

N^i is the counting rate observed with a Pd absorber of thickness t_i. N^0 is the counting rate with no absorber at all. A_i is a correction term accounting for the detection of 21.2-keV palladium x rays, $\epsilon'=0.961$ is the detector efficiency for 21.2-keV photons, $\mu'=14.4$ cm^2/gm is the total absorption coefficient of palladium for 21.2-keV photons and $\mu_x{}^K=47.1$ cm^2/gm is the K-absorption coefficient for 25.3-keV photons. A_i becomes negligible for $t_i\gtrsim 0.004$ in. Ω_{SA}, Ω_{AD}, and Ω_{SD} are the solid angles subtended by the absorber at the source position by the detector at the absorber position, and by the detector at the source position, respectively. $e^{-\mu_\gamma\rho t}$ was measured directly by absorbing the x rays in 0.004 in. of Pd absorber located 1.5 cm below the source, and performing a regular absorption experiment with various palladium absorbers 0.002 in. thick, 1 in. diam, placed 21 cm below the source, the midpoint between source and detector. $\mu_\gamma=10.5$ cm^2/gm was obtained for $\rho t=0.0604$ gm/cm^2. As a check on this experiment, the absorption coefficients at 23.8 keV were also measured for Al, Cu, Sn, Mo, and Ag. These results, agreed in all cases to within 5% with the NBS data[9] on Al, Cu, Mo, and Sn, with the Handbook of Chemistry and Physics tables[11] on Al, Cu, Mo, Ag, and Sn, with A. J. Bearden's measurements[12] on Al, Cu, and Sn, and with R. D. Deslattes[6] absorption data on Al, Cu, Sn, Mo, Pd, and Ag indicating that no unaccounted-for scattering distorted the absorption data. Scattering by the air and surrounding objects was measured with the source in position and a lead plug at the absorber position, and all counting rates were appropriately corrected. The source was attached with paper tape to a lucite holder with a 1-in. hole in the middle to prevent back scattering. Deslattes' value for[6] μ_x, $\mu_x=56.1$ cm^2/gm was used. The product ρt of density and thickness of the square foils used was obtained by weighing the foils and dividing the mass by the area.

The ratios $R=1.64\pm0.10$, 1.68 ± 0.17, 1.63 ± 0.25, and 1.64 ± 0.34 were obtained with one, two, three, and four 0.002-in. palladium absorbers, respectively, combining to an average of $R=1.65\pm0.08$. The error in R was obtained by adding quadratically the errors arising from the estimated 5% uncertainties in μ_x and in μ_γ.

If the effect on R arising from a 2.3% Sn^{113} contamination is considered (the complete expression is given in the Appendix) the ratios R for the four absorption experiments performed become 1.96, 2.04, 1.72, and 1.48, respectively. These four values are definitely not consistent with each other, which is a strong indication that the source is not contaminated with Sn^{113}.

In the second experiment the Mössbauer effect of the 23.8-keV transition is measured with and without a palladium absorber.[13] Without palladium the unresolved x rays reaching the detector simply add to the background. With 0.005 in. of palladium between the source and the absorber, mostly γ rays are transmitted and the number of Sn x rays and Pd x rays reaching the detector is negligible. Under these conditions, the ratio R can be written in terms of ϵ_{Pd} and ϵ_0, the observed Mössbauer absorption effects with and without palladium absorber, $[N(v=0)]_{\mathrm{Pd}}$ and $[N(v=0)]_0$ the counting rates with and without palladium absorber and the source at rest with respect to the absorber, S the counting rate due to scattering by the surroundings, $\mu_x{}^A$ and $\mu_\gamma{}^A$ the atomic absorption coefficients of x and γ rays in the resonant absorber of thickness t and density ρ:

$$R=e^{(\mu_x{}^A-\mu_\gamma{}^A)\rho t}\left[\frac{\epsilon_{\mathrm{Pd}}}{\epsilon_0}\frac{1-[S/[N(v=0)]]_0(1-\epsilon_0)}{1-[S/[N(v=0)]]_{\mathrm{Pd}}(1-\epsilon_{\mathrm{P}})}1-\right].$$

The experiment was carried out at room temperature with a SnO_2 source in pressed powder form, containing

[10] M. E. Rose, *Internal Conversion Coefficients* (North-Holland Publishing Company, Amsterdam, 1958).

[11] *Handbook of Chemistry and Physics*, edited by C. D. Hodgman (Chemical Rubber Publishing Company, Cleveland, Ohio, 1962).

[12] A. J. Bearden, Ph.D. thesis, Johns Hopkins University, 1958 (unpublished).

[13] We wish to express our thanks to Professor R. Herber and H. Stoeckler for carrying out these measurements and preparing the SnO_2 ceramic samples.

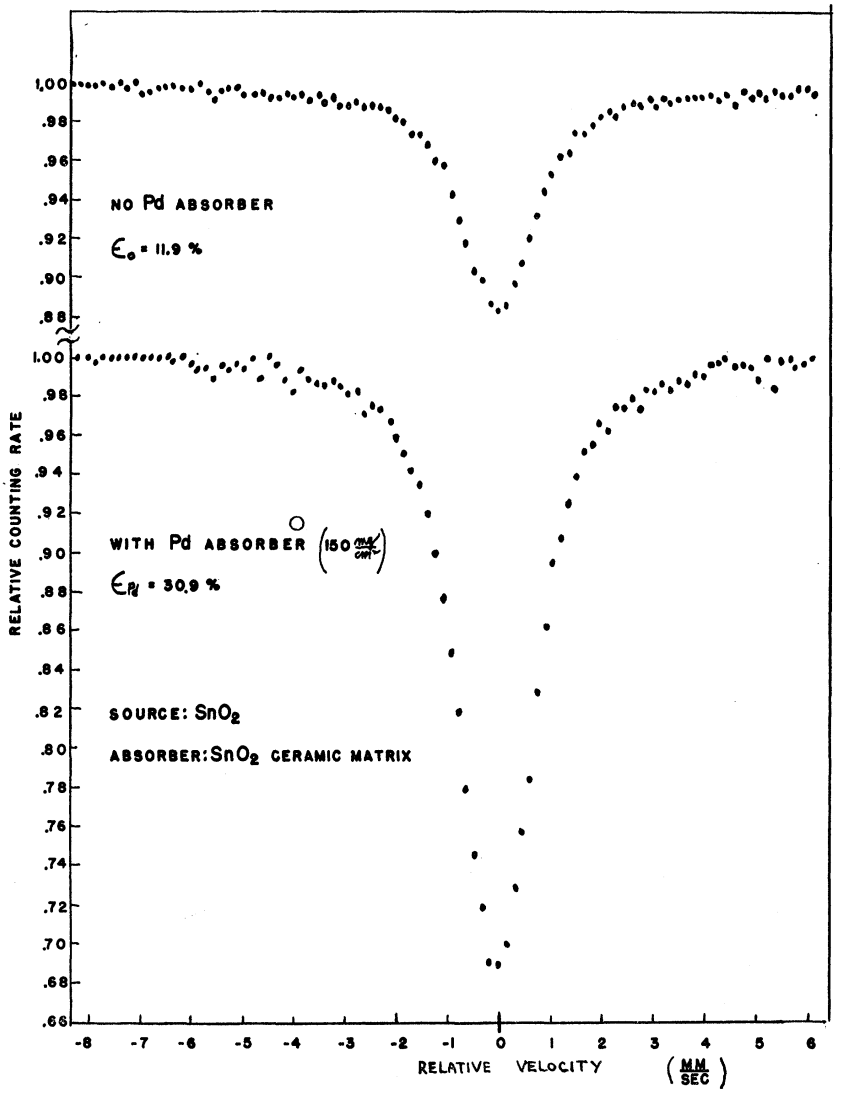

FIG. 3. Mössbauer absorption spectra obtained at room temperature, with a SnO_2 source (9 mg/cm² of Sn) in pressed powder form and a SnO_2 absorber (35 mg/cm² of Sn) in a ceramic matrix.

9 mg/cm² of Sn, and a SnO_2 absorber in a ceramic lattice (10% SnO_2, 5% Na_2CO_3, and 85% B_2O_3) containing 35 mg/cm² of Sn. Typical absorption spectra are shown in Fig. 3. These measurements yielded a value $R=1.65\pm0.06$.

Since the two alternate methods of determining R are more readily subject to possible systematic errors, only the coincidence results were used in the final evaluation of α and $[K/(L+M+\cdots)]_{M4}$.

IV. CONCLUSION

The final results $\alpha=5.2\pm0.3$ and $[K/(L+M+\cdots)]_{M4}=0.42\pm0.02$ are in good agreement with the theoretical values for pure $M1$ and $M4$ transitions, $\alpha(M1)=5.3$ and $[K/(L+M)]_{M4}=0.38$, interpolated from Rose's[10] tables of internal conversion coefficients. The $[K/(L+M+\cdots)]_{M4}$ ratio is also remarkably close to Hill's result obtained with an electron spectrometer. However, the present value for total conversion coefficient differs sufficiently from the previously accepted values to cause a very drastic change, both in the absolute value, and in the temperature dependence of the Debye temperature Θ and of the Debye-Waller factor $2W$ for tin metal absorbers obtained from some of the Mössbauer absorption measurements. In fact, in view of the fact that a small percentage error in α corresponds to a very large error in $2W$, considerably more precision is required in the measurement of α before the Mössbauer effect data on Sn^{119} can be interpreted in terms of lattice vibrations.

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APPENDIX

The expressions derived in Sec. II relating R and $[K/(L+M+\cdots)]_{M4}$ to the measured single and coincidence rates can be expanded to include the effect of Pd x rays and In^{113} x rays in the following manner: No palladium absorber:

$$N_{1,2}=N_{1,2x}+N_{1,2\gamma}+N_{1,2x}(\mathrm{In})=N_0/(R+1)[R+\epsilon_\gamma/\epsilon_x+nf(1+\alpha)(\epsilon''/\epsilon_x)]\epsilon_x\Omega_{1,2}$$

$$N_c=N_{1x}\Omega_2\epsilon_\gamma[N_{0\gamma}/(N_{0\gamma}+N_{0c})]+N_{1\gamma}\Omega_2\epsilon_x[K/(K+L+M+\cdots)]_{M4}\omega_K$$

$$(N_c/N_1N_2)_0=\omega_K\frac{2(R+1)(\epsilon_\gamma/\epsilon_x)}{N_0[R+\epsilon_\gamma/\epsilon_x+nf(1+\alpha)(\epsilon''/\epsilon_x)]^2[1+(L+M+\cdots)/K]_{M4}}.$$

With palladium absorber:

$$N_1=N_{1\gamma}+N_{1x}(\mathrm{Pd})+N_{1x}(\mathrm{In})=N_{0\gamma}\epsilon_\gamma\Omega_1e^{-\mu_\gamma\rho t}[1+RAe^{\mu_\gamma\rho t}[\Omega_{SA}\Omega_{AD1}/\Omega_1](\epsilon'/\epsilon_\gamma)+nf(1+\alpha)e^{-(\mu''-\mu_\gamma)\rho t}(\epsilon''/\epsilon_\gamma)],$$

$$N_2=N_{2\gamma}+N_{2x}+N_{2x}(\mathrm{Pd})+N_{2x}(\mathrm{In})=N_{0\gamma}\epsilon_\gamma\Omega_2[1+R(\epsilon_x/\epsilon_\gamma)+RA'[\Omega_{SA}\Omega_{AD2}/\Omega_2](\epsilon'/\epsilon_\gamma)+nf(1+\alpha)(\epsilon''/\epsilon_\gamma)],$$

$$N_c=N_{1\gamma}\Omega_2\epsilon_x[K/(K+L+M+\cdots)]_{M4}\omega_K+N_{1\gamma}\Omega_2\epsilon'[K/(K+L+M+\cdots)]_{M4}\omega_KA'\Omega_{SA}\Omega_{AD2}/\Omega_2$$
$$+N_{1x}(\mathrm{Pd})\Omega_2\epsilon_\gamma[N_{0\gamma}/(N_{0\gamma}+N_{0c})],$$

$$(N_c/N_1N_2)_{\mathrm{Pd}}=\omega_K\frac{[1+A'\Omega_{SA}\Omega_{AD2}/\Omega_2+Ae^{\mu\gamma\rho},\Omega_{SA}\Omega_{AD1}/\Omega_1]}{N_0[1+(L+M+\cdots)/K]_{M4}}$$
$$\times\frac{1}{[1+RAe^{\mu_\gamma\rho t}(\Omega_{SA}\Omega_{AD1}/\Omega_1)+nf(1+\alpha)e^{-(\mu''-\mu_\gamma)\rho t}][1+[RA'/(R+1)](\Omega_{SA}\Omega_{AD2}/\Omega_2)+nf(1+\alpha)/(1+R)]},$$

where Ω_{SA}, Ω_{AD1}, Ω_{AD2} and $\Omega_{1,2}$ are the solid angles subtended by the Pd absorber at the source, by the detector #1 at the absorber, by the detector #2 at the absorber, and by detectors #1 or #2 at the source, respectively. A was described in Sec. III. A' is a similar term, but for the case when the Pd x rays go back through the absorber towards detector #2:

$$A_i'=\omega_K[1-e^{-(\mu_x+\mu')\rho t_i}]\mu_x/(\mu_x+\mu').$$

$\epsilon''=0.965$ is the detection efficiency for 24.2-keV In x rays and $\mu''=10.3$ cm²/gm is the palladium absorption coefficient for these x rays. $n=1.4$ is the number of In x rays emitted per disintegration of Sn^{113} and f is the ratio of Sn^{113} to Sn^{119} activities present in the source.

The γ and x radiations Compton scattered into detector #2 are in coincidence with the same radiations as the primary γ and x rays detected in that counter. For this reason, the scattered radiation can be readily included in the above formula by just replacing the geometric solid angle Ω_2 in the expressions for N_{2x} and $N_{2\gamma}$, and in the corresponding expression for N_c, by an effective, slightly larger solid angle Ω_2'. Ω_2' is different for x and γ rays, because of the difference in the Compton scattering cross section and in the detection efficiencies for the two radiations. It is very difficult to make these corrections with any precision, but their contribution to $(N_c/N_1N_2)_{\mathrm{Pd}}$ was estimated, and since it is less than 1%, it was not included in the above formula.

The expression for R derived in Sec. III for the case of critical absorption measurements can also be generalized to include the effect of In^{113} x rays.

$$R=\frac{(e^{-\mu_\gamma\rho t_i}-N^i/N^0)(\epsilon_\gamma/\epsilon_x)+(e^{-\mu''\rho t_i}-N^i/N^0)nf(1+\alpha)(\epsilon''/\epsilon_x)}{N^i/N^0-e^{-\mu_x\mu t_i}-(\Omega_{SA}\Omega_{AD}/\Omega)A_i(\epsilon'/\epsilon_x)}.$$